# AKHIMO 2018 Problem 1

**Problem 1.** Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers defined by

$$x_1 = 2018, \qquad \frac{x_{n+1}}{x_n} = 2018 + \frac{2018}{n} \quad \text{for} \quad n \geq 1.$$

Find the limit

$$\lim_{n\to\infty} \frac{\log_2 n}{n - \log_{2018} x_n}.$$

# AKHIMO 2018 Problem 2

**Problem 2.** Let $f : [0,2] \to \mathbb{R}$ be a continuous and non-decreasing function. Prove that

$$\int_{-1}^{1} f(x^2+1)\,\mathrm{d}x \geq \int_{0}^{2} f(x)\,\mathrm{d}x.$$

When does the equality hold?

# AKHIMO 2018 Problem 3

**Problem 3.** Let $n \geq 2$ be an integer. Denote by $A_1, A_2, \ldots, A_n \in M_2(\mathbb{C})$ all solutions of the equation

$$X^n = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}.$$

Prove that

$$\sum_{k=1}^{n} \operatorname{Tr}(A_k) = 0.$$

Here $M_2(\mathbb{C})$ is the set of $2 \times 2$ complex matrices and $\operatorname{Tr}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d$.

# AKHIMO 2018 Problem 4

**Problem 4.** Let $n > 2018$ be an integer. Suppose that there are subsets $A_1, A_2, \ldots, A_n$ of the set $\{1, 2, \ldots, n\}$ such that

(i) $|A_k| = 2018$ for all $k \in \{1, \ldots, n\}$,

(ii) $|A_i \cap A_j|$ is divisible by 3 for all $1 \leq i < j \leq n$,

where $|S|$ is the number of elements of the set $S$. Prove that $n$ is an even integer.

# AKHIMO 2018 Problem 5

**Problem 5.** Let $s$ be a positive integer. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence given by

$$a_1 = 0, \quad a_2 = 2s, \quad a_3 = 3, \quad a_{n+3} = sa_{n+1} + a_n \quad \text{for} \quad n \geq 1.$$

Prove that for any prime number $p$, the number $a_p$ is divisible by $p$.